Systematic Theoretical Search for Dibaryons in a Relativistic Model

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A relativistic quark potential model is used to do a systematic search for quasi-stable dibaryon states in the u, d, and s three flavor world. Flavor symmetry breaking and channel coupling effects are included and an adiabatic method and fractional parentage expansion technique are used in the calculations. The relativistic model predicts dibaryon candidates completely consistent with the nonrelativistic model.

I. INTRODUCTION

Since Jaffe predicted the first dibaryon, the H particle [1], there have been many experimental and theoretical efforts to search for dibaryons. Up to now, there are no experimentally well established dibaryon states except the deuteron. Theoretically almost all QCD models, including lattice QCD, predict that there should be dibaryon states [2]. However no model has yet achieved an acceptable level of quantitative reliability. Either the model Hamiltonian is oversimplified or the model Hilbert space is rather restricted, or both. We developed a nonrelativistic model, which we termed the quark delocalization color screening model(QDCSM) [3–5]. The model Hamiltonian and Hilbert space are reasonable enough to yield a qualitatively correct N-N interaction including both repulsive core and intermediate range attraction and to fit all of the four (IJ)=(01) (10) (00) and (11) channel phase shifts qualitatively. This model reproduces an almost correct deuteron state and verifies dynamically that there are two kinds of dibaryons [6]. One kind is a loosely bound, low spin (J), two baryon state with only slight quark delocalization, of which the deuteron is prototypical. Another kind is a tightly bound, high spin, six quark state with significant quark delocalization; here, the $d^\star(IJ{=}03)$ [6] is a prime example. (We do not discuss, however, potential dibaryons with more involved internal structure, such as those with non-valence internal structure, such as would correspond to NNπ bound states [7], or with orbital excitations of the valence quarks [8].)

It is believed that systems of light $(u, d$ and $s)$ quarks require relativistic dynamics. Therefore it is better to have a relativistic model calculation to check if those dibaryon states predicted by the nonrelativistic model(QDCSM) are robust against relativistic effects. A relativistic quark potential model(LAMP) [9] has been developed for this purpose and the fractional parentage technique developed for the nonrelativistic dibaryon calculation has been extended to the relativistic case as well [10]. Together, these make a systematic relativistic quark potential model dibaryon search feasible and the results are reported here.

II. THE RELATIVISTIC QUARK POTENTIAL MODEL

We neglect the small current quark mass for u and d and treat them as massless, $m_u = m_d = m = 0$. For a single baryon, we assume a relativistic Hamiltonian with a scalar linear confinement [9],

$$H(B) = \sum_{i=1}^{3} H_i + \sum_{i<j} H_{ij} \tag{1}$$

$$H_i = \vec{\alpha}_i \cdot \vec{p}_i + \beta_i(m_i + V(r_i)) \tag{2}$$

$$V(r_i) = k^2(r_i - r_0) \tag{3}$$

$$H_{ij} = g_s(m_i)g_s(m_j)\frac{\vec{\lambda}_i \cdot \vec{\lambda}_j}{4}\vec{\alpha}_i \cdot \vec{\alpha}_j A e^{-\nu(\vec{r_i}-\vec{r_j})^2} \tag{4}$$

where α_i, β_i are Dirac matrices, $\vec{\lambda}_i$ is the color SU(3) generator, m_i is the quark mass. $V(r)$ is a phenomenological confinement potential, which we assume is a Lorentz scalar and r_i is the modulus of the three-coordinate of the i-th quark relative to the origin. The color Coulomb interaction due to gluon exchange is assumed to be absorbed in the form of Eq.(3) even though it is a Lorentz vector. This is done to simplify the numerical calculation. The parameters k^2=0.9 GeV/fm and $r_0 = 0.57$ fm are fixed by the condition that the eigenenergy E of the following Dirac equation [12].

$$H_i \psi_{\sigma_i}(\vec{r_i}) = E \psi_{\sigma_i}(\vec{r_i}) \tag{5}$$

is one third of the average of the nucleon (N) and Delta(Δ) masses, $E = \frac{1}{6}(N + \Delta)$, for massless quarks. The lowest energy wavefunction solution is:

$$\psi_{\sigma_i}(\vec{r_i}) = \begin{pmatrix} \phi_u(r_i) \\ -i\vec{\sigma_i} \cdot \hat{r}_i \phi_l(r_i) \end{pmatrix}$$

$$\phi_u(r) = P_u(kr) e^{-k^2 r^2/2}$$

$$\phi_l(r) = P_l(kr) e^{-k^2 r^2/2} \tag{6}$$

$$P_u(kr) = 1 + 0.04287 kr + 0.00457 (kr)^2 + \cdots$$

$$P_l(kr) = -kr(0.46330 - 0.08767 kr + \cdots)$$

H_{ij} is the color magnetic interaction due to gluon exchange, assumed to keep the form of single gluon exchange. A Gaussian gluon propagator is adopted for the color magnetic interaction to simulate the confinement property of a color gluon on the one hand and to simplify the numerical calculation on the other hand. $\nu = (\frac{0.2GeV}{\hbar c})^2 = 1.0$ fm^{-2} is chosen to be about the size of a hadron. The combination $g_s^2(m = 0)A = -1.350$ GeV is fixed by $N - \Delta$ mass difference, where A is the matrix element of the spatial part of the (color) current-current interaction matrix element. Note that the quark-gluon effective coupling is assumed to vary with a scale related to the quark mass.

Note that, having so-modified the form of single gluon exchange, this two-body-interaction Hamiltonian could equally well be taken to represent the effects of instantons [11]. The color structure, limited range, and quark-mass-dependent effective strength are all also features found in the effective quark-quark interactions produced by the propagation of quarks in the presence of instantons. The diquark correlations expected in instanton models are thus implicitly implemented by this H_{ij}.

The single baryon ground state wave function (WF) is assumed to be a product of single quark WFs,

$$\psi_B(123) = \chi_c(123) \left[\prod_{i=1}^{3} \psi_{\sigma_i}(\vec{r_i}) \eta_{f_i}(i) \right]_{SIJ} \tag{7}$$

where $\chi_c(123)$ is a three quark color singlet state. η_{f_i} is the single quark flavor WF. $[\,]_{SIJ}$ means the individual quark spin-flavors are coupled to total baryon strangness S, isospin I and spin J.

The strange quark mass $m_s = 307$ MeV is determined by the $\Lambda - N$ mass difference. (Although a factor of about two larger than conventional in quark models, this difference can be traced to a virial theorem factor of one-half which arises from the use of a Dirac scalar potential.) The ratio, $g_s(m_s)/g_s(0)$, is determined by the overall fit of the octet and decuplet baryon masses. The fitted baryon masses and the value of $g_s(m_s)/g_s(0)$ are shown in Table 1.

Table 1. Baryon masses. $g_s(m_s)/g_s(0)$=1/1.26

	N	Δ	Λ	Σ	Ξ	$\Sigma^\star$	$\Xi^\star$	Ω
exp.	939	1232	1116	1193	1318	1385	1533	1675
theor.	939	1232	1116	1182	1333	1376	1527	1683

For a two baryon system, we change the Hamiltonian slightly from that given in Eq.(1): i now runs from 1 to 6 and the confinement potential is replaced by

$$V_2(r_i) = \min_{\mathrm{J}=1,2} \{ V(\,|\,\vec{r}_i - \vec{R}_J\,|\,)\,\} \tag{8}$$

where $\vec{R}_1$ and $\vec{R}_2$ are two baryon 'centers' [9] and $V(r_i)$ is given in Eq.(3). This confinement potential is devised to simulate the effect that, as the two baryons are brought close together, the quark matter density in between them will increase gradually, causing the QCD vacuum to change (also gradually) from nonperturbative to perturbative. As one quark is removed to a distance large compared to the separation of the two centers, the confining potential should be the same as for an isolated baryon. These effects are achieved by truncating the value of the confining potential on the midplane between the two baryon centers, $\vec{R}_1$ and $\vec{R}_2$.

The two baryon WF is assumed to be a linear combination of the following Dirac cluster WFs,

$$\Psi_{\alpha K}(q^6) = \mathcal{A}[\psi(B_1)\psi(B_2)]_{SIJ} \tag{9}$$

$$\begin{aligned} \psi(B_1) &= \chi_{c_1}(123)\left[\prod_{i=1}^{3}\psi^L_{\sigma_i}(\vec{r}_i)\eta_{f_i}(i)\right]_{S_1 I_1 J_1} \\ \psi(B_2) &= \chi_{c_2}(456)\left[\prod_{j=4}^{6}\psi^R_{\sigma_j}(\vec{r}_j)\eta_{f_j}(j)\right]_{S_2 I_2 J_2} \end{aligned} \tag{10}$$

$$\begin{aligned} \psi^L_{\sigma_i}(\vec{r}_i) &= (\psi_{\sigma_i}(\vec{r}_i - \vec{R}_1) + \epsilon(x)\psi_{\sigma_i}(\vec{r}_i - \vec{R}_2))/N(x) \\ \psi^R_{\sigma_j}(\vec{r}_j) &= (\psi_{\sigma_j}(\vec{r}_j - \vec{R}_2) + \epsilon(x)\psi_{\sigma_j}(\vec{r}_j - \vec{R}_1))/N(x) \end{aligned} \tag{11}$$

$$\begin{aligned} x &= |\vec{R}_1 - \vec{R}_2| \\ N^2(x) &= 1 + \epsilon^2(x) + 2\epsilon(x)\left\langle \psi_{\sigma_i}(\vec{r}_i - \vec{R}_1) | \psi_{\sigma_i}(\vec{r}_i - \vec{R}_2)\right\rangle \end{aligned} \tag{12}$$

$\mathcal{A}$ is the normalized antisymmetrization operator. $\alpha = (SIJ)$ and K represent the other quantum numbers related to B_1 and B_2, χ_{c_1}, and χ_{c_2} are both the three quark color singlet states.

A physical two baryon state with quantum number $\alpha = (SIJ)$ is

$$\Psi_\alpha(q^6) = \sum_K C_{\alpha K} \Psi_{\alpha K}(q^6) \tag{13}$$

The summation K runs over all the possible two baryon states consisting of the octet and decuplet baryons listed in Table I. This is sufficient as it spans the space of relevant asymptotic states.

The six quark Hamiltonian is first diagonalized in the Dirac cluster basis space given by Eq.(13). To simplify the six quark Hamiltonian matrix element calculations, a relativistic extension of the fractional parentage expansion method is used [10]. The one-body and two-body matrix elements are calculated by a program developed in the preliminary relativistic dibaryon calculations [9,13].

The lowest eigen-energy obtained in the diagonalization is x and $\epsilon(x)$ dependent, and $\epsilon(x)$ is varied at each x to obtain the minimum. The difference between the minimum at x and infinite separation is assumed to be the effective potential of the two baryons. (The minimum at $x = 3$ fm is already negligibly different from values at larger separation.) We expect that spurious effects such as center-of-mass (c.m.) motion largely cancel out in the effective potential due to the process described above. Therefore, we have neither subtracted the c.m. motion energy separately for a single baryon nor for the two baryons together. Following Ref. [6], we use the following expression to calculate the dibaryon masses,

$$M_\alpha(q^6) = (M_1 + M_2)_\alpha + \min[V_\alpha(x)] + \frac{3}{4\mu_\alpha} \frac{\hbar^2}{x_0^2} \tag{14}$$

where $\min[V_\alpha(x)]$ is the minimum of the effective interaction $V_\alpha(x)$, and $(M_1 + M_2)_\alpha$ is the channel weighted experimental two baryon mass,

$$(M_1 + M_2)_\alpha = \sum_K |C_{\alpha K}|^2 < \Psi_{\alpha K}(q^6) | \Psi_{\alpha K}(q^6) > (M_1 + M_2)_{\alpha K}, \tag{15}$$

at the point x_0 at which the minimum occurs.

A zero point oscillation energy of $\frac{3\hbar^2}{4\mu_\alpha x_0^2}$ has been taken into account as explained in Ref. [6]. Even though the internal motion of the quarks is relativistic, we assume the relative motion of the two baryon centers still can be approximated as a nonrelativistic oscillation around the equilibrium separation x_0, where μ_α is the weighted reduced channel mass,

$$\mu_\alpha = \sum_K |C_{\alpha K}|^2 < \Psi_{\alpha K}(q^6) | \Psi_{\alpha K}(q^b) > \left(\frac{M_1 M_2}{M_1 + M_2} \right)_{\alpha K}. \tag{16}$$

This model has been used to calculate the binding energies of ^{4}He and ^{3}He, for which the reasonable values 19 MeV (^{4}He), 3.8 MeV (^{3}He) have been obtained without any adjustable parameters [9].

III. RESULTS

A systematic dibaryon search has been done in the u, d and s three flavor world. Both channel coupling and flavor symmetry breaking effects have been taken into account. The interesting dibaryon states are listed in Tables 2a,b.

Table 2a lists 'Model A' results, where an SU(3) flavor symmetric WF is used and the one body energy difference due to $m_s \neq m$ have also been neglected. The two body matrix elements are assumed to obey the (empirically consistent) relations

$$\langle sl|H_{ij}|sl\rangle = \frac{2}{3}\langle ll|H_{ij}|ll\rangle$$
$$\langle ss|H_{ij}|ss\rangle = \frac{4}{9}\langle ll|H_{ij}|ll\rangle \tag{17}$$

$\langle ll|H_{ij}|ll\rangle = A$ is calculated using the massless quark WF (Eq.6) operator matrix element multiplied by the overall constant such that $g_s^2(m=0)$A= -1.350 GeV. The notation sl(ss) means strange-nonstrange (strange-strange) two body matrix elements. Table 2b lists the 'Model B' results where the one-body energy for the u, d and s quarks and the two-body matrix elements are both calculated with the WF obtained from Eq.(5) with m=0 and m_s = 307 MeV, respectively. The overall constants, $g_s^2(0)$A= -1.350 GeV and $g_s(0)g_s(m_s)$A and $g^2(m_s)$A appear in $\langle ll|H_{ij}|ll\rangle$, $\langle sl|H_{ij}|sl\rangle$ and $\langle ss|H_{ij}|ss\rangle$ respectively to obtain the full flavor dependence of the two body matrix elements. The value $g_s(m_s) = g_s(0)/1.26$ determined by the ground state baryon masses has been used.

In principle we should use an SU(3) flavor symmetry breaking WF to study the flavor symmetry breaking effects. We use these two model results to show the uncertainty of the flavor symmetry breaking effect in our model. The largest difference is for the $\alpha = (-600)$ state where the model A and B results have 140 MeV difference. Except for this extreme strangeness case, the largest difference is about 60 MeV corresponding to $\alpha = (-3\frac{1}{2}2)$. The channel coupling effects are similar to the nonrelativistic case.

The most prominent feature is that the relativistic model yields dibaryon masses in the whole u, d, s three flavor world quite similar to those of the nonrelativistic model. This is shown in Table 3, where NA, NB, RA and RB stand for nonrelativistic color screening version A and B [6] and the relativistic model A and B (with channel coupling and flavor symmetry breaking, *ccb*) results, respectively. The similarity is true also for states omitted from these tables. This might be considered surprising as the relativistic and nonrelativistic model are not only different in kinematics but also in the details of the confinement mechanism. We take this similarity of results as a mutual confirmation of the stability to the model details of the estimated dibaryon masses in these two models.

Comparing the relativistic (R) and nonrelativistic (N) model estimates on the dibaryon masses more carefully, one finds there are differences fluctuating from case to case.

In the nonstrange sector, the R model masses are higher than the N ones. In the R model, the deuteron is unbound and the model α = (003) state is ~60 MeV higher than the N one. For the strangeness -1 sector, the N and R models give almost the same dibaryon masses.

Beginning at strangeness -1 and generally increasing with increasing strangeness, the R model gives larger binding. Except for the α = (-220) state, where the R model mass is 10 MeV higher than the N model mass, in all the other cases, R masses are smaller than N masses, ranging from -25 MeV for the α = (-400) state to -220 MeV for the α = (-600) and α = (-3$\frac{1}{2}$2) states. The α = (-600) state is a special example, where the two versions of the R model

themselves have a 140 MeV difference, while all others differ by less than ±60 MeV. However it is interesting to note that skyrmion model practitioners have predicted the same strangeness -6 dibaryon candidate [14].

This general trend of differences between the R and N models may be understood in the following way: In the N model fit to the octet and decuplet baryon masses, the theoretical octet masses are larger than experiment while the decuplet masses are smaller. This indicates that the color magnetic interaction decreases (theoretically) more than warranted from the nonstrange to the strange quark case, and is due to the relatively large strange quark mass needed for the overall fit to the data. In addition, the R model calculations for dibaryon masses use variational upper bounds for the kinetic energies, and so should lead to overestimates in all cases. Finally, the two versions of the R model differ from each other more with increasing strangeness since they treat the flavor symmetry breaking in the one-body matrix elements differently, as described earlier.

Excluding a few special cases, the mass differences, among all of these model estimates are in the neighborhood of ±50 MeV, on average. We take this as the uncertainty of the present dibaryon mass estimate, using the R and N models altogether.

Although our primary interest has been in the α=(003), the R model predicts an H particle state very similar to that of Jaffe [1], which has already been extensively searched for. Experimental results [15] on doubly strange hypernuclei cast doubt on its existence as a bound state, (although the issue of the relative binding energy of an H in a nucleus vs that of two Λ's has not, to our knowledge, yet been addressed seriously). In the R model, it is ~60 MeV lower than the $\Lambda\Lambda$ threshold, and its wave function is almost a pure flavor singlet as is Jaffe's. However, since the N model finds the H to be unbound by 35 MeV, this state is clearly sensitive to dynamical details. Therefore, in our models, it cannot definitively theoretically be concluded that the H exists as a bound state.

A preliminary relativistic calculation of the α=(003) state was reported in Ref. [13], where this state is denoted $d^\star$. This $\Delta\Delta$ state has the largest binding energy in the u, d and s three flavor world. Because it is a nonstrange state, the mass estimated by our model (both relativistic and nonrelativistic) should be more reliable. Even if we take the highest estimated mass, it is still 15 MeV lower than NN$\pi\pi$ threshold. Therefore the $d^\star$ is quite possibly a narrow dibaryon resonance. For comparison, a small-hard-core radius, large NNρ-coupling meson exchange model obtained a similar state [16].

The strange sector also has a few dibaryon candidates. We note especially the spin 3 states, which all have a larger predicted binding energy than the d*. They have been discussed in Ref. [4]; we refer the reader to that discussion.

As to the question of prospects for experimental searches: There have, of course, been many efforts to search for Jaffe's H. For the $d^\star$ state, a preliminary estimate of the $\pi^\pm d \to \pi^\pm d^\star$ production cross section is of order 0.1 μb [13]. This may be too small to be detected within the large general pion scattering background. A similar reaction [17]

$$\pi^- +^3 He \to n + d^\star \to n + n + n + \pi^+ \tag{18}$$

and its charge conjugate reaction

$$\pi^+ +^3 H \rightarrow p + d^* \rightarrow p + p + p + \pi^- . \tag{19}$$

may be more favorable for the detection of $d^\star$ production. Here one can suppress the background by measuring the 'spectator' $n(p)$ and the emitted $\pi^+(\pi^-)$ in coincidence. Unfortunately, pion beams of the required energy range and intensity do not seem to be currently available at any accelerator facility.

On the one hand, coupling to the $I = 0$, ${}^3D_3\, NN$ channel is expected to be small, obviating a search by this method. However on the other hand, an analysis by Lomon [18] suggests that the channel coupling may not be small enough to avoid conflict with results from phase shift analyses of scattering data already available, even though the experimental points are widely spread in energy.

Finally, we note that a recent calculation by Wong [19] of a proton induced reaction similar to the pion induced one

$$p +^2 H \rightarrow p + d^\star \rightarrow p + p + p + \pi^- \tag{20}$$

may produce sufficient signal for observation, again using detection of the π^- to suppress backgrounds. Such an experiment has been proposed at TRIUMF (Canada) by S. Yen (Spokesman for E772).

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Table 2a. Model A results:

An SU(3) flavor symmetric WF is used and the one body

energy difference due to $m_s \neq m$ has been neglected.

s_0 is the value of x (in $fm.$) at which the minimum

energy of the dibaryon state occurs.

SIJ		M_α	V_α	ϵ	s_0	Threshold
001	scs	1902	-8	0.2	1.4	1878(NN)
	ccs	1898	-8	0.2	1.5	
010	scs	—	0	0.0	3.0	1878(NN)
	ccs	—	0	0.0	3.0	
003	scs	2143	-349	1.0	1.3	2464($\Delta\Delta$)
						2158(NN$\pi\pi$)
-1$\frac{1}{2}$3	scb	2309	-334	1.0	1.3	2617($\Delta\Sigma^*$)
	ccb	2309	-334	1.0	1.3	2335(N$\Lambda\pi\pi$)
-1$\frac{3}{2}$0	scb	2145	-9	0.2	1.6	2132(NΣ)
	ccb	2145	-9	0.2	1.6	
-200	scb	2110	-230	0.4	0.8	2231($\Lambda\Lambda$)
	ccb	2173	-242	0.3	0.7	
-202	scb	2279	-235	0.6	1.1	2472(NΞ^*)
	ccb	2371	-240	0.5	1.0	2397(N$\Xi\pi$)
-213	scb	2469	-322	1.0	1.3	2765($\Delta\Xi^*$)
	ccb	2476	-317	1.0	1.3	2690($\Delta\Xi\pi$)
-220	scb	2396	-12	0.2	1.5	2386($\Sigma\Sigma$)
	ccb	2396	-12	0.2	1.5	
-3$\frac{3}{2}$3	scb	2616	-312	1.0	1.3	2904($\Delta\Omega$)
	ccb	2634	-302	1.0	1.3	2788($\Lambda\Xi^*\pi$)
-3$\frac{3}{2}$1	scb	2494	-49	0.4	1.2	2511($\Sigma\Xi$)
	ccb	2496	-50	0.4	1.2	
-3$\frac{1}{2}$2	scb	2405	-255	0.6	1.0	2611(NΩ)
	ccb	2481	-252	0.4	0.9	2574($\Lambda\Xi\pi$)
-3$\frac{1}{2}$1	scb	2350	-132	0.6	1.0	2434($\Lambda\Xi$)
	ccb	2456	-139	0.4	0.9	
-400	scb	2622	-33	0.3	1.5	2636($\Xi\Xi$)
	ccb	2623	-33	0.3	1.5	
-600	scb	3232	-131	0.5	1.4	3345($\Omega\Omega$)

scs – single channel only, flavor symmetry

ccs – with channel coupling, flavor symmetry

scb – single channel only, broken flavor symmetry

ccb – with channel coupling, broken flavor symmetry

Table 2b. Model B results:

One-body energy and the two-body matrix elements are both calculated with the WF solutions for m=0 and $m_s = 307$ MeV.

SIJ		M_α	V_α	ϵ	s_0	Threshold
001	scs	1902	-8	0.2	1.4	1878(NN)
	ccs	1898	-8	0.2	1.5	
010	scs	—	0	0.0	3.0	1878(NN)
	ccs	—	0	0.0	3.0	
003	scs	2143	-349	1.0	1.3	2464($\Delta\Delta$)
						2158(NN$\pi\pi$)
-1$\frac{1}{2}$3	scb	2293	-354	0.9	1.2	2617($\Delta\Sigma^*$)
	ccb	2293	-354	0.9	1.2	2335(N$\Lambda\pi\pi$)
-1$\frac{3}{2}$0	scb	2145	-9	0.1	1.6	2132(NΣ)
	ccb	2145	-9	0.1	1.6	
-200	scb	2071	-269	0.4	0.8	2231($\Lambda\Lambda$)
	ccb	2171	-274	0.2	0.6	
-202	scb	2246	-277	0.5	1.0	2472(NΞ^*)
	ccb	2332	-286	0.4	0.9	2397(N$\Xi\pi$)
-213	scb	2432	-363	0.9	1.2	2765($\Delta\Xi^*$)
	ccb	2439	-358	1.0	1.2	2690($\Delta\Xi\pi$)
-220	scb	2393	-22	0.3	1.3	2386($\Sigma\Sigma$)
	ccb	2393	-22	0.3	1.3	
-3$\frac{3}{2}$3	scb	2564	-374	0.8	1.1	2904($\Delta\Omega$)
	ccb	2581	-363	0.7	1.1	2788($\Lambda\Xi^*\pi$)
-3$\frac{3}{2}$1	scb	2459	-98	0.3	1.0	2511($\Sigma\Xi$)
	ccb	2462	-99	0.3	1.0	
-3$\frac{1}{2}$2	scb	2346	-326	0.6	0.9	2611(NΩ)
	ccb	2420	-327	0.4	0.8	2574($\Lambda\Xi\pi$)
-3$\frac{1}{2}$1	scb	2296	-197	0.6	0.9	2434($\Lambda\Xi$)
	ccb	2420	-210	0.3	0.8	
-400	scb	2591	-71	0.3	1.3	2636($\Xi\Xi$)
	ccb	2591	-71	0.3	1.3	
-600	scb	3093	-281	0.5	1.1	3345($\Omega\Omega$)

Second column labels are same as in Table 2A.

Table 3. The Comparison of the masses of six-quark system between different versions of model.

SIJ	NA: $\nu = 1.6 fm^{-2}$			NB: $\nu = 0.6 fm^{-2}$			RA			RB		
	M_α	ϵ	s_0	M_α	ϵ	s_0	M_α	ϵ	s_0	M_α	ϵ	s_0
001	1874	0.2	1.3	1879	0.3	1.3	1898	0.2	1.5	1898	0.2	1.5
010	1885	0.1	1.6	1890	0.2	1.4	—	0.0	3.0	—	0.0	3.0
003	2084	1.0	1.2	2073	1.0	1.2	2143	1.0	1.3	2143	1.0	1.3
$-1\frac{1}{2}3$	2292	1.0	1.1	2285	1.0	1.1	2309	1.0	1.3	2293	0.9	1.2
$-1\frac{3}{2}0$	2131	0.2	1.4	2127	0.2	1.4	2145	0.2	1.6	2145	0.1	1.6
-200	2255	1.0	0.6	2257	1.0	0.6	2173	0.3	0.7	2171	0.2	0.6
-202	2456	1.0	0.8	2455	1.0	0.8	2371	0.5	1.0	2332	0.4	0.9
-213	2508	1.0	1.0	2495	1.0	1.1	2476	1.0	1.3	2439	1.0	1.2
-220	2386	0.2	1.3	2382	0.2	1.3	2396	0.2	1.5	2393	0.3	1.3
$-3\frac{3}{2}3$	2711	1.0	0.9	2697	1.0	1.0	2634	1.0	1.3	2581	0.7	1.1
$-3\frac{3}{2}1$	2509	0.4	1.0	2506	0.4	1.0	2496	0.4	1.2	2462	0.3	1.0
$-3\frac{1}{2}2$	2669	1.0	0.6	2670	1.0	0.6	2481	0.4	0.9	2420	0.4	0.8
$-3\frac{1}{2}1$	2560	1.0	0.6	2562	1.0	0.6	2456	0.4	0.9	2420	0.3	0.8
-400	2632	0.2	1.1	2630	0.3	1.1	2623	0.3	1.5	2591	0.3	1.3
-600	3375	1.0	0.6	3376	1.0	0.6	3232	0.5	1.4	3093	0.5	1.1